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TITLE: GENERAL CP PROPERTIES OF NEUTRINO MASS EIGENSTATES

AUTHOR(S): S. Peter Rosen, T-DO

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Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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GENERAL CP PROPERTIES OF NEUTRINO MASS EIGENSTATES

S. P. Rosen
T-Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ABSTRACT

We show that the mass eigenvectors of the neutrino mass matrix have definite CP quantum numbers whether or not CP is conserved, and we examine the conditions under which a mixture of even and odd CP eigenstates will occur.

We have heard several discussions of "pseudo-Dirac" neutrinos[1] and other possibilities[2] in which neutrino flavor eigenstates are linear combinations of Majorana mass eigenstates with opposite CP eigenvalues. Because of their opposite CP quantum numbers, these mass eigenstates interfere destructively[3] in the amplitude for no-neutrino double beta decay and thereby give rise to an "effective Majorana mass" which is much smaller than the mean mass of the electron-neutrino. In this talk, I shall consider how eigenstates of CP emerge from the general neutrino mass matrix, and the conditions under which they may have opposite CP eigenvalues.

Let me begin the discussion by examining the case of one generation. For convenience I work with a Dirac neutrino field ψ, and its charge conjugate:

$$\psi_c = C\tilde{\bar{\psi}} \equiv \gamma_2 \psi^* \quad . \tag{1}$$

Under parity ψ will be taken to transform according to the rule

$$\psi \overset{P}{\rightarrow} i\rho\, \gamma_4 \psi \quad , \quad (\rho = \pm 1) \tag{2}$$

where the imaginary element is deliberately inserted in anticipation of the well-known fact that Majorana neutrinos have imaginary parity[3]. I now form fields

$$\chi\,(\pm) \equiv \frac{1}{\sqrt{2}}\,(\psi \pm \psi_c) \tag{3}$$

which, by virtue of eqs. (1) and (2), are eigenstates of both C and CP; in the former case their eigenvalues are ($\pm$ 1) respectively and in the latter ($\pm\, i\rho$). These are the fields that represent Majorana neutrinos with opposite CP quantum numbers.

The mass matrix can be written down explicitly as

$$\{M_D\, \bar{\psi}\psi + M_D\, \bar{\psi}_c\, \psi_c + M_M\, \bar{\psi}\psi_c + M_M^*\, \bar{\psi}_c\, \psi\} \tag{4}$$

or in matrix form as

$$[\bar{\psi},\ \bar{\psi}_c] \begin{bmatrix} M_D & M_M \\ M_M^* & M_D \end{bmatrix} \begin{bmatrix} \psi \\ \psi_c \end{bmatrix} \equiv \bar{\Psi}\, M\, \Psi \tag{5}$$

In equation (4), the first term is a Dirac mass term of exactly the same kind as occurs in the quark and charged-lepton mass matrices, and the second term represents the mass of the charge conjugate field explicitly. The third and fourth terms are the so-called Majorana mass terms coupling the field and its charge conjugate.

Hermiticity of the mass term in eq. (4) ensures that $\mathcal{M}_D$ is real, but it places no restriction on $\mathcal{M}_M$. CP invariance, however, would require that $\mathcal{M}_M$ be real. Whether or not CP is conserved, the mass matrix $\mathcal{M}$ in eq. (5) is Hermitian

$$\mathcal{M}^{+} = \mathcal{M} \tag{6}$$

and its complex conjugate has the additional property

$$C\,\mathcal{M}^{*}\,C = \mathcal{M} \tag{7}$$

where

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

Equation (6) guarantees that the eigenvalues of $\mathcal{M}$ are real, and equation (7) implies that the eigenvectors of $\mathcal{M}$ have the form:

$$\mathcal{M} \begin{bmatrix} a \\ \pm a^{*} \end{bmatrix} = \lambda \begin{bmatrix} a \\ \pm a^{*} \end{bmatrix} \tag{9}$$

Consequently the matrix U which diagonalizes $\mathcal{M}$ must be of the general form:

$$U = \begin{bmatrix} a & b \\ a^{*} & -b^{*} \end{bmatrix}$$

$$U^{+}\mathcal{M}U = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{10}$$

It now follows that the eigenvectors of the mass matrix are:

$$\Phi = U^{+}\,\Psi \equiv \begin{bmatrix} a^{*}\psi + a\psi_c \\ b^{*}\psi - b\psi_c \end{bmatrix} \tag{11}$$

The upper component of Φ has even CP eigenvalues $(+\, i\rho)$ and the lower odd CP $(-\, i\rho)$. Thus in the one-generation case, the Majorana mass eigenstates have opposite CP quantum numbers, irrespective of CP conservation.

Let us now generalize this result to the case of N generations by making the following substitutions for the fields ψ and ψ_c:

$$\psi \to \Psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \psi_N \end{pmatrix}; \; \psi_c \to \Psi_c \equiv \begin{pmatrix} \psi_{1c} \\ \psi_{2c} \\ \cdot \\ \cdot \\ \psi_{NC} \end{pmatrix} \tag{12}$$

The elements of $\mathcal{M}$ must be replaced by (N x N) matrices with the properties:

$$\mathcal{M}_D \to M_D = M_D^+; \; \mathcal{M}_M \to M_M = \tilde{M}_M \tag{13}$$

(+ denotes Hermitian conjugate, and ~ is transpose), and $\mathcal{M}$ itself becomes a (2N x 2N) Hermitian matrix:

$$\mathcal{M} \to \hat{\mathcal{M}} \equiv \begin{bmatrix} M_D & M_M \\ M_M^* & \tilde{M}_D \end{bmatrix} = (\hat{\mathcal{M}})^+ \tag{14}$$

Without loss of generality, we can assume that each component of Ψ transforms with the same phase under parity[4] ($\psi_\ell \to i\rho\, \gamma_4\, \psi_\ell$ for $\ell = 1, 2, \ldots N$), and so $\hat{\mathcal{M}}$ will be a real matrix if CP should be conserved.

The matrix C of eq. (8) also becomes a (2N x 2N) matrix:

$$C \to C_N \equiv \begin{bmatrix} 0 & I_N \\ I_N & 0 \end{bmatrix} \tag{15}$$

where I_N is an (N x N) unit matrix. It is not difficult to show that in addition to the Hermiticity of $\hat{\mathcal{M}}$, the analogue of the property in eq. (7) is also preserved:

$$C_N\, (\hat{\mathcal{M}})^*\, C_N = \hat{\mathcal{M}} \tag{16}$$

Therefore the matrix which diagonalizes $\hat{\mathcal{M}}$ must be of the form:

$$U_N = \begin{bmatrix} A & B \\ A^* & -B^* \end{bmatrix} \tag{17}$$

where A and B are (N x N) submatrices (not necessarily unitary), and the mass eigenvectors are:

$$\Phi_N = U_N^+ \begin{bmatrix} \Psi \\ \Psi_c \end{bmatrix} = \begin{bmatrix} A^*\, \Psi + A\, \Psi_c \\ B^*\, \Psi - B\, \Psi_c \end{bmatrix} \tag{18}$$

Again the upper components have even CP and the lower components odd CP.

When CP is conserved, M_D and M_M are both real, symmetric matrices and the diagonal form of $\hat{N}$ is given by:

$$\begin{bmatrix} A^{+}(M_D + M_M)A & , & 0 \\ 0 & , & B^{+}(M_D - M_M)B \end{bmatrix} \tag{19}$$

The transformation matrices A and B are now real and orthogonal. From this we see that the CP even eigenvectors are associated with the eigenvalues of $(M_D + M_M)$ and the CP odd eigenvectors with $(M_D - M_M)$.

Our final generalization is to introduce the chiral decomposition of the fields into Ψ_R, the right-handed multiplet, and Ψ_L, the left-handed one. Thus we make the substitutions:

$$\Psi \to \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix} ; \quad \Psi_C \to \begin{pmatrix} \Psi_{CL} \\ \Psi_{CR} \end{pmatrix} \tag{20}$$

and replace the (N x N) Dirac and Majorana matrices by (2N x 2N) ones:

$$M_D \to \begin{bmatrix} 0 & D \\ D^{+} & 0 \end{bmatrix} \quad ; \quad M_M \to \begin{bmatrix} R & 0 \\ 0 & L \end{bmatrix} \tag{21}$$

where R and L are symmetric (N x N) matrices. The zeros in eq. (21) correspond to the fact that mass terms can only connect left-handed fields with right-handed ones and vice versa.

Because of the structure of M_D and M_M in eq. (21), it is easy to show that the eigenvalues of $(M_D - M_M)$ must be equal and opposite to those of $(M_D + M_M)$. Since we identify only eigenvectors with positive mass eigenvalues as physical states, it follows that the positive eigenvalues of $(M_D + M_M)$ will have eigenvectors with even CP. The negative eigenvalues of $(M_D + M_M)$ become positive ones for $(M_D - M_M)$ and hence will be associated with odd CP eigenvectors.

In conclusion we see that in general the neutrino mass eigenvectors have definite CP eigenvalues, but the precise mixture of even and odd ones will depend on the detailed structure of the eigenvalue spectrum of $(M_D + M_M)$.[5]

References

(1) T. Kotani, these proceedings.

(2) S. P. Rosen, these proceedings.

(3) B. Kayser, these proceedings.

(4) Any field ψ_R which transforms under parity with a phase $(-\rho)$ can always be replaced by $(\gamma_5\psi_R)$ which transforms with phase $(+i\,\rho)$.

(5) For further details see S. P. Rosen, Phys. Rev. D (to be published).